Quiz 6 (10pts)
Math 214 Section Q1 Winter 2010

Your name:________________________________ ID#:______________

Please, use the reverse side if needed.

1.(5 pts) Find the vector projection of $\mathbf{u} = \langle 3, -1, 2 \rangle$ onto $\mathbf{v} = \langle 3, 4, 0 \rangle$, and the scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$.

Solution.

Vector projection of $\mathbf{u}$ onto $\mathbf{v}$:

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} = \frac{9-4}{3^2+4^2}\langle 3, 4, 0 \rangle = \frac{1}{5}\langle 3, 4, 0 \rangle = \langle \frac{3}{5}, \frac{4}{5}, 0 \rangle$$

Scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$

$$\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|} = \frac{9-4}{\sqrt{3^2+4^2}} = 1.$$

2.(5 pts) Find the area of the triangle with vertices $P_1(1, 1, 1)$, $P_2(5, 3, 3)$, and $P_3(2, 1, 2)$.

Solution.

$\overrightarrow{P_1P_2} = \langle 4, 2, 2 \rangle$, $\overrightarrow{P_1P_3} = \langle 1, 0, 1 \rangle$.

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 2 & 2 \\ 1 & 0 & 1 \end{vmatrix} = 2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}.$$

Area of the parallelogram determined by $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$:

$$|\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}| = \sqrt{2^2 + (-2)^2 + (-2)^2} = \sqrt{12} = 2\sqrt{3}.$$

Area of the triangle is $\sqrt{3}$.